

Status of the Hadronic Contribution to the Muon $(g - 2)$ Value

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ABSTRACT

With the recent interest in the measured and standard model values of the muon anomalous magnetic moment, a_μ , some confusion has arisen concerning our knowledge of the hadronic contribution to a_μ . In the dispersion integral approach to hadronic vacuum polarization effects, low energy contributions must be evaluated from data or in a model-dependent approach tested by data. At higher energies perturbative QCD has been used, sometimes in conjunction with data. The history of such evaluations is reviewed, and the prospects for further improvement are discussed. We conclude that not all published evaluations are on an equal footing or up-to-date. One must critically examine which, and how much information went into each analysis in order to determine which are more complete, and reliable.

1 Introduction

With the new result from the Muon $(g - 2)$ Collaboration [1], comparison with the standard model prediction for a_μ has received renewed scrutiny. This result represents the third measurement by the E821 collaboration [2, 3], and the first to approach the part per million (ppm) level of precision. That level of precision permits a stringent new test of the standard model, and a search for physics beyond it. However, before conclusions can be drawn, it is necessary to have a reliable standard model calculation with sub-ppm accuracy for comparison with the experimental number. In this

paper, the status of QED and weak contributions are briefly reviewed and then we focus on the hadronic contributions. After an overview of all published evaluations of the hadronic contribution since 1985, we discuss several in detail, with an eye towards assessing both reliability, and whether the calculation is current or has been superseded as a point for comparison with experiment by more up-to-date studies.

The standard model prediction for $a_\mu \equiv (g_\mu - 2)/2$ consists of three parts,

$$a_\mu(\text{theory}) = a_\mu(\text{QED}) + a_\mu(\text{Hadronic}) + a_\mu(\text{Weak}). \quad (1)$$

Taking the value of α from the electron ($g - 2$) [4], yields the total QED contribution [5, 6]

$$a_\mu(\text{QED}) = 116\,584\,705.7(2.9) \times 10^{-11}, \quad (2)$$

which is dominated by the first-order (Schwinger) term $\alpha/2\pi$ but is calculated (or estimated) through $\mathcal{O}(\alpha/\pi)^5$. The uncertainty is very small and should not play a role in comparisons with experiment.

The weak contribution through second order is [7, 8, 9, 10]

$$a_\mu(\text{weak}) = 152(4) \times 10^{-11} \quad (3)$$

contributing about 1.3 ppm of a_μ (assuming a 150 GeV Higgs mass). The 3-loop electroweak leading logs have been estimated to contribute $+0.5 \times 10^{-11}$. That small effect is safely covered by the uncertainty in eq. (3).

Although QED and electroweak effects now appear to be well under control, there have been some sizeable shifts in their predicted values over the years due to error corrections and improved higher order calculations. In Table 1, we illustrate changes in the theoretical prediction for a_μ that have occurred since a summary was given in 1990 [14].

While the QED and weak contributions are well described perturbatively, the hadronic contribution cannot be completely calculated from perturbative QCD, but must instead be determined in part by using data from the cross-section for [15, 16, 17, 18, 19, 20, 21]

$$e^+e^- \rightarrow \text{Hadrons} \quad (4)$$

in conjunction with a dispersion integral. The uncertainty in those data, particularly at low energies, largely determines the error in the Standard Model prediction for a_μ . More recently, data from hadronic τ decays have also been used along with information from perturbative QCD at relatively low energies [22, 23, 24] to reduce uncertainties.

<i>Quantity</i>	<i>1990 Value</i> ($\times 10^{11}$)	<i>2001 Value</i> ($\times 10^{11}$)	<i>Change ($\times 10^{11}$)</i>
a_{μ}^{QED}	116 584 695.5(5.4)	116 584 705.7(2.9)	+10.2
a_{μ}^{Had} (vac. pol 1)	7 068(59)(164)	6 924(62)	-144
a_{μ}^{Had} (vac. pol 2)	-90(5)	-100(6)	-10
a_{μ}^{Had} (light by light)	49(5)	-85(25)	-134
a_{μ}^{EW} (1 loop)	195(10)	195	0
a_{μ}^{EW} (2 loop)	—	-43(4)	-43
a_{μ}^{SM} (total)	116 591 918(176)	116 591 597(67)	-321

Table 1: Improvements in the theoretical calculation of a_{μ} from 1990 [14] to 2001. The major shifts were primarily due to errors in the earlier calculations, new calculations of higher order effects, improved $e^+e^- \rightarrow$ hadrons and tau data, and additional utilization of perturbative QCD.

2 The Hadronic Contributions

In this section we describe the basic issues and summarize the various evaluations of the hadronic contributions to the muon anomaly, including the higher-order 3-loop effects. Then we discuss in some detail one of the data-driven analyses in order to illustrate the relative importance of the various energy regions in the evaluation of the leading hadronic contribution, $a_{\mu}(\text{Had}; 1)$, and the main sources of the errors on it. Finally, we discuss some specific details of several of the calculations. In particular, it is hard to ignore the recent assertion [25] that we currently have only modest knowledge of a_{μ} , and all calculations are of equal merit. In this latter section we will refute the claims made in that paper.

2.1 The Leading Hadronic Contribution

The leading hadronic contribution comes from the vacuum polarization diagram shown in Fig. 1(a). Because the loop integration involves low energy scales near the muon mass, the contributions cannot be calculated from perturbative QCD alone. At higher loop momenta perturbative QCD becomes applicable, and it is common in the evaluations of $a_{\mu}(\text{Had}; 1)$ to effectively switch from data to QCD at some energy scale.

Since the leading order hadronic vacuum polarization contribution is derived primarily from data, its value continues to be improved by new e^+e^- cross-section measurements. In that regard, the CMD2 collaboration at the VEPP2M collider in

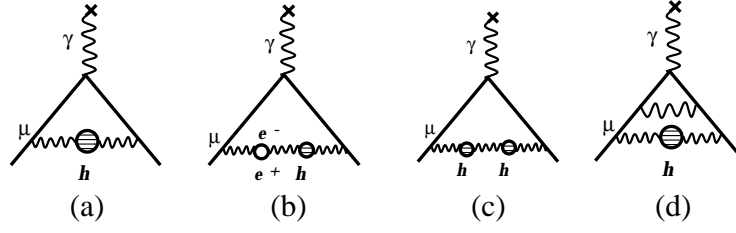


Figure 1: (a) The leading hadronic vacuum polarization contribution. (b-d) Examples of higher order $(\alpha/\pi)^3$ contributions derived from the hadronic vacuum polarization.

Novosibirsk has collected substantial data from the 2π threshold up to $\sqrt{s} = 1.4$ GeV, and is preparing a publication on these data [11]. Similarly, the BES collaboration in Beijing has recently measured $R(s)$ from $\sqrt{s} = 2.0$ GeV up to 5 GeV [12]. Data from hadroproduction (see Fig. 2(a)) can be related through a dispersion relation to the first order hadronic vacuum polarization of Fig.1(a).

A second way in which the hadronic contribution can be improved is through hadronic τ decays to vector final states. The relevant diagram is shown in Fig. 2(b) where the weak charged current can be related to the isovector part of the electromagnetic current in Fig. 2(a) through the conserved vector current (CVC) hypothesis plus the additional requirements of isospin conservation and the absence of second-class currents (which is the case for the Standard Model). While the weak τ decay proceeds through both vector and axial vector weak-currents, the final states with an even number of pions (even G -parity) are the ones relevant for $(g - 2)$, since decays to these final states go exclusively through the vector current if there are no second class currents. Of course, isospin violating effects of order a few percent must be properly accounted for.

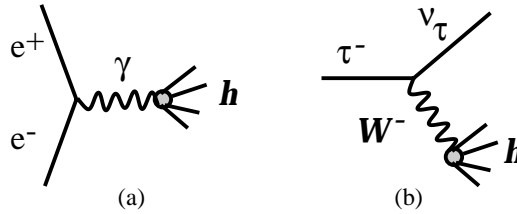


Figure 2: (a). The hadroproduction process which enters the dispersion relation. (b) Hadronic τ decay.

The measurements of

$$R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (5)$$

are used as input to the dispersion relation,

$$a_\mu(\text{Had}; 1) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s) \quad (6)$$

where the kernel is given by

$$K(s) = \frac{3s}{m_\mu^2} \left\{ x^2 \left(1 - \frac{x^2}{2}\right) + (1+x)^2 \left(1 + \frac{1}{x^2}\right) \left[\ln(1+x) - x + \frac{x^2}{2} \right] + \frac{1+x}{1-x} x^2 \ln x \right\} \quad (7)$$

with

$$x = \frac{1-\beta}{1+\beta}, \quad \beta = \sqrt{1 - \frac{4m_\mu^2}{s}}. \quad (8)$$

Since there is a factor of s^{-2} in the dispersion relation, values of $R(s)$ at low energies dominate the calculation of $a_\mu(\text{Had}; 1)$.

Here, we should point out that R is often not directly measured experimentally. In those cases where the cross-section for e^+e^- is determined using some other normalization, careful subtractions for initial state radiation, vacuum polarization etc. have to be applied to the data whereas most such effects would naturally cancel in the ratio R . In Table 2 the published evaluations of $a_\mu(\text{Had}; 1)$ since 1985 are given.

In carrying out the dispersion integrals, there are two ways of integrating the data. In B85 and KNO85 the e^+e^- data were fit to models such as a Gounaris-Sakurai resonance parameterization [26] or extensions thereof, and these theoretical curves were then utilized to obtain $R(s)$ for the dispersion integral. EJ95 [19] were the first to employ a model-independent trapezoidal integration of data. BW96 [20], also used a trapezoidal integration and took into account the correlations between the systematic errors in the data. Below we employ one of these two model independent evaluations, which are in excellent agreement with one another, as a benchmark to discuss other evaluations of the hadronic contribution.

2.2 Higher Order Hadronic Contributions

The higher-order (3-loop) hadronic contributions come from the hadronic vacuum polarization diagrams in Fig.1(b-d) and the hadronic light-by-light scattering shown in Fig.3

The higher order hadronic contributions in Fig1(b)–(d) were most recently evaluated by Krause [29]. The set of diagrams represented by Fig. 1(d) give a dominant

<i>Ref.</i>	$a_\mu(\text{Had}; 1)$ ($\times 10^{11}$)	$a_\mu(\text{Had}; 1)$ <i>in ppm</i>	<i>Comments</i>
B85 [15]	6840 (110)	58.6 (0.9)	primarily e^+e^- data
KNO85 [16]	7070 (180)	60.6 (1.54)	primarily e^+e^- data
CLY85 [17]	7100 (115)	60.9 (0.9)	QCD, theory and some e^+e^-
MD90 [18]	7048(115)	60.5 (1.0)	e^+e^- and QCD
MD90 [18]	7052(76)	60.5 (0.65)	e^+e^- and QCD
EJ95 [19]	7024 (153)	60.3 (1.4)	primarily e^+e^- data
BW96 [20]	7026 (160)	60.3 (1.4)	primarily e^+e^- data
AY95 [21]	7113 (103)*	60.0 (0.9)	QCD, theory and some e^+e^-
ADH98 [22]	6950 (150)	59.6 (1.29)	primarily e^+e^- data
ADH98 [22]	7011 (94)	60.1 (0.8)	$e^+e^- + \tau$ data
DH98a [23]	6951 (75)	59.6 (0.6)	e^+e^- , τ and perturbative QCD at energies ($E > 1.8$ GeV)
DH98b [24]	6924 (62)	59.4 (.53)	e^+e^- , τ and perturbative QCD and QCD sum rule constraints at low energy

Table 2: The first-order hadronic vacuum polarization contribution to $(g-2)$ obtained by a number of different authors. (Earlier evaluations with much larger uncertainties are not exhibited.)

*The value of $a_\mu(\text{Had}; 1)$ given in the abstract of [21](hep-ph/9509378) does not agree with the value given in the conclusions section of text. Perhaps this confusion was corrected in the 1998 published version which is not available to us. We assume that the value quoted in [25] is the value to take from [21]. Similarly, in [17] a second method for evaluating the ρ contribution gave a somewhat smaller result 7045 which is not illustrated in the table.

negative contribution. Collectively, Krause found $-101(6) \times 10^{-11}$. That value was slightly updated by Alemany, Davier and Höcker [22] to

$$a_\mu(\text{Had}; 2) = -100(6) \times 10^{-11} \quad (0.86 \pm 0.05)\text{ppm} \quad (9)$$

which supersedes the earlier value [16]. The difference between the earlier evaluation[16] and the more recent calculation is attributed to the use of the full kernel function in the new calculation[29], and more up-to-date hadronic data.

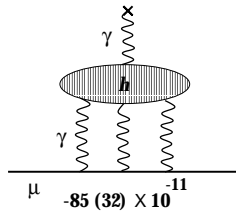


Figure 3: The hadronic light-by-light scattering contribution.

2.2.1 Hadronic Light-by-Light Contribution

The hadronic light-by-light contribution shown in Fig.3 was first calculated by KNO85 [16]. Unlike the hadronic contributions discussed above, this contribution cannot be determined from data, so one is dependent on a model calculation. Einhorn [30] pointed out that the vector meson dominance model used in KNO85 did not satisfy the Ward-Takahashi identities which are required by electromagnetic gauge symmetry. Furthermore, part of the calculation was in error.

There have been three recent calculations of the hadronic light-by-light scattering contribution, one by Hayakawa, Kinoshita and Sanda (HKS) [31], one by Bijnens, Pallante and Prades (BPP) [32], and a follow-up improved calculation by Hayakawa and Kinoshita (HK) [33]. Along the way, low energy theorems were developed by Hayakawa¹ for the (p -wave) $V^0 - \pi$ scattering amplitude ($V^0 = \rho^0, \omega$ or ϕ) [34].

The situation seems to have converged as far as is possible without a first principles or lattice calculation of the four-point function. Since the light-by-light contribution currently can only be obtained from calculation, the give and take between two groups, and the mutual checking of the other's calculation was invaluable in resolving the magnitude and uncertainty on this correction. In the final analysis, it may be the uncertainty of this contribution which provides the ultimate limitation on the standard model prediction for $a_\mu(\text{Had})$. Indeed, subtle cancellations in the calculation deserve further study. Fortunately, the current level of uncertainty appears suitable for the final goals of E821.

Following [22], we take the average of (BPP) and (HK),

$$a_\mu(\text{Had; lol}) = -85(25) \times 10^{-11} \quad (-0.72 \pm 0.21) \text{ ppm} \quad (10)$$

The total higher order hadronic correction from (9) and (10) is

$$a_\mu(\text{Had; Higher order}) = -185 \pm 26 \times 10^{-11}$$

¹In Ref. [34] the behavior in the limit of low pion momentum is studied, and it is shown that the s -wave component must vanish in the chiral limit under quite general assumptions.

3 The Hadronic Contribution Compared with a_μ^{exp}

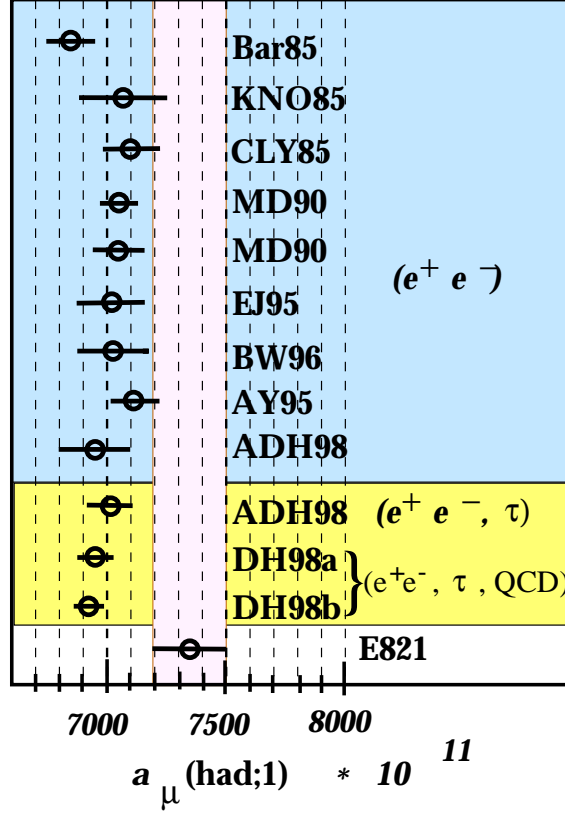


Figure 4: A comparison of the calculated values of $a_\mu(\text{Had}; 1)$ along with the difference between the measured value of a_μ less the QED, weak and higher order hadronic corrections.

The recent result reported by E821 is

$$a_\mu^{\text{exp}} = (116\,592\,020 \pm 160) \times 10^{-11} \quad (11)$$

where the systematic and statistical errors have been added in quadrature. When combined with the previous measurements, one gets

$$\langle a_\mu \rangle_{\text{exp}} = (116\,592\,023 \pm 151) \times 10^{-11} \quad (12)$$

Since the QED and electroweak contributions are not in question, we can subtract those calculated values from the experimental number. The resulting quantity represents the hadronic contribution plus any contribution from new physics. Next, we subtract off the higher order hadronic contributions given above and obtain

$$a_\mu(\text{Had}; 1) + a_\mu(\text{New?}) = 7350(153) \times 10^{-11}. \quad (13)$$

This number is to be compared with the values of $a_\mu(\text{Had}; 1)$ illustrated above in Table 2. A graphical comparison is given in Fig. 4.

This compilation represents a completely uncritical selection of published evaluations. Nevertheless, all evaluations agree with each other and deviate in varying degrees with the value suggested by experiment. We discuss some of these in detail below. Because of the common data sets which went into most of these evaluations, the errors on these evaluations of $a_\mu(\text{Had}; 1)$ are highly correlated.

4 The Different Evaluations of $a_\mu(\text{Had}; 1)$

We now give a detailed critique of the evaluations of $a_\mu(\text{Had}; 1)$. Rather than starting the discussion with B85, KNO85, or CLY85, all of which used a parameterization for the ρ resonance we first consider the model-independent analyses of EJ95 and BW96.

While the analyses of e^+e^- data by EJ95, BW96 and ADH98 obtain essentially identical final results, there are differences on several points between the three analyses. In EJ95 two results are quoted. These are given in Table 3a, and Table 3b of ref. [19]. It is the former which is directly comparable to the other analyses. The so-called “renormalization group improvement” result quoted in EJ95 (Table 3b) represents an effort by those authors to incorporate the higher order contribution of Fig. 1(b) into the lowest order hadronic vacuum polarization. Unfortunately, that approach is invalid and should be discounted. The fact that it is quoted in the abstract of EJ has caused some recent confusion [25]. The authors (EJ) themselves have not continued to advocate the Renormalization Group improved approach, but instead have accepted the conventional method of computing higher order hadronic corrections separately as done by Krause.[29]

Although both EJ95 and BW96 do a model independent analysis, there is a difference in how errors and experimental data are combined. Up to 2 GeV, the data have traditionally been published as exclusive cross sections. Above 2 GeV they are published as the inclusive cross section ratio $R(s)$. It is necessary to integrate over energy, sum over modes (below 2 GeV), and combine results from different detectors. EJ95 computed an error weighted average of $R(s)$ over all experiments, but in so doing, they lost the information on correlations between the errors over energy.

BW96 combined the data in a way which permitted the correlations to be included, and they also added a scale factor when combining results from different experiments, thereby handling experiments which do not agree in a conservative manner.

While the EJ95 method gives up the information on the correlations over energy, and BW96 explicitly includes them, the final results are so close that there is no practical difference between the two methods or results. We conclude that the correlations

are not important. Furthermore, when the better data which are now available from BES and other e^+e^- facilities are included in the future, the issue becomes irrelevant.

4.1 The Contributions to $a_\mu(\text{Had}; 1)$ from Different Energy Regions

It is important to understand the source of the errors on $a_\mu(\text{Had}; 1)$ how they have recently been improved and will be further improved over time as new data become available. In Table 3, the contributions to $a_\mu(\text{Had}; 1)$ from different energy regions in the dispersion integral found in 1996 by Brown and Worstell are listed with their uncertainties.

Energy Region (GeV)	$a_\mu(\text{Had}; 1)$ ($\times 10^{11}$)	% of $a_\mu(\text{Had}; 1)$	δ_{tot} ($\times 10^{11}$)	δ_{frac} (ppm)
$\sigma(e^+e^- \rightarrow \text{Hadrons}) \sqrt{s} < 1.4$	6113.32	87	149.97	1.29
$\sigma(e^+e^- \rightarrow \text{Hadrons}) 1.4 \leq \sqrt{s} \leq 2.0$	324.66	4.6	24.96	0.21
$R(s)^a 2.0 \leq \sqrt{s} \leq 3.1$	283.74	4.0	35.51	0.30
$R(s)^b 2.0 \leq \sqrt{s} \leq 2.6$	204.8	2.9	31.47	0.27
$R(s)^b 2.6 \leq \sqrt{s} \leq 3.1$	78.93	1.1	13.99	0.12
J/ψ (6 states)	90.47	1.3	9.69	0.08
Υ (6 states)	1.09	-	0.13	-
QCD $3.1 \leq \sqrt{s} < \infty$	213.01	3.0	3.71	0.03
Subtotal $\sqrt{s} < 3.1 + J/\psi + \Upsilon$ (no QCD)	6813.28	97	160.22	1.37
Subtotal $\sqrt{s} < 1.4$	6113.32	87	149.97	1.29
Subtotal $1.4 < \sqrt{s} < 3.1$	608.40	8.7	55.51	0.48
Subtotal $\sqrt{s} > 1.4$ (Includes QCD)	912.97	13	56.48	0.48
Total had1	7026.29	100	160.25	1.37

Table 3: The 1996 contributions to $a_\mu(\text{Had}; 1)$ from the various energy regions with their total errors. The systematic errors are twice the statistical errors in the region from threshold to 2.0 GeV and from 2.6 to 3.1 GeV. From 2.0 to 2.6 GeV the systematic and statistical errors are about equal. (This table is based on Table IX. from Ref. [20]).

The largest contribution comes from threshold to 1.4 GeV. However, in the context of this model-independent analysis, one sees that if the error from the 2π threshold to 1.4 GeV were eliminated completely, one would still have been left with an error of $\delta_{\text{tot}} = 56.5 \times 10^{-11}$, or 0.48 ppm. The recent Aleph τ -decay data have been used to significantly improve the region below 1.4 GeV, and inclusion of the full LEP and

CLEO τ data samples along with the new e^+e^- Novosibirsk data will improve things further.

The region between 2.0 and 2.6 GeV had been particularly problematic. In 1996 there were only two experiments, BCF and $\gamma\gamma 2$, which have comparable total errors, but $\gamma\gamma 2$ has a large systematic error. This small region alone contributed an uncertainty of ~ 0.27 ppm to $a_\mu(\text{Had}; 1)$, while the entire region from 2.0 to 3.1 GeV contributed ~ 0.31 ppm error. (Without correlations these errors add in quadrature.) With the new data from BES [12], which agree with the QCD evaluation by Davier and Höcker, it appears that the problems in this energy region are solved and the uncertainty is significantly reduced. Hence, more current studies of $a_\mu(\text{Had}; 1)$ have justifiably smaller uncertainties than earlier efforts.

4.2 Brief Overview of the Evaluations of $a_\mu(\text{Had}; 1)$

Before giving a more detailed discussion of the evaluations listed in Table 2, we provide a brief perspective. Our point of view is that the knowledge of the hadronic contribution to a_μ is an evolving topic, that earlier analyses which represented the state of the art at one time, become outdated when new data and improved evaluations become available.

When B85 and KNO85 made their analyses, there were many unpublished data from Orsay which were not included. By 1995 these data were mostly published, so we believe EJ95, BW96 and ADH98 supersede the earlier studies. We defer the discussion of CLY85 and AY95 to the next section, but note in passing that one of the goals of MD90 was to improve the estimate of the theoretical errors presented in CLY85.

The values of $a_\mu(\text{Had}; 1)$ obtained by EJ95, BW96 and ADH98 are quite consistent. It is interesting to note the large improvement obtained by ADH98 by the inclusion of the τ -decay data. From Fig. 4 and Table 2 one can see that the addition of the τ -decay data raised the value of $a_\mu(\text{Had}; 1)$ slightly, but to a value quite consistent with the earlier analyses. The use of QCD for $\sqrt{s} > 1.8$ GeV by DH98a, along with e^+e^- and τ -decay data lowered the value of $a_\mu(\text{Had}; 1)$ and significantly reduced the error, since the QCD prediction was systematically below the existing data (discussed above) in the energy region 2-3 GeV. This QCD prediction, which was done in advance of the recently reported $R(s)$ measurement at Beijing [12], is in excellent agreement with the new data, which do not seem to suffer from the systematic problems of the older BCF and $\gamma\gamma 2$ data [13].

This excellent agreement between the QCD calculation and the new $R(s)$ data gives one confidence in the evaluation of $a_\mu(\text{Had}; 1)$ presented in DH98a. Since the input to this evaluation contains much more data than the earlier evaluations, and the theory input seems to be justified by the new Beijing data, we believe that one must at least take DH98a as the best evaluation of $a_\mu(\text{Had}; 1)$ up to that point. In

DH98b, the same authors use QCD sum rule constraints at lower energies to further improve the uncertainty on their evaluation of $a_\mu(\text{Had}; 1)$, and it is this last value which is used in Ref. [1] to compare the experimental and theoretical evaluations. That final improvement is perhaps a more controversial improvement. However, we note that DH98a and DH98b do not significantly differ.

To illustrate the degree of improvement in $a_\mu(\text{Had}; 1)$, we give in Table 4 contributions from different energy regions as obtained by Davier and Höcker.[24] Although

\sqrt{s} (GeV)	$a_\mu(\text{Had}; 1) \times 10^{11}$
$2m_\pi - 1.8$	6343 ± 60
1.8–3.7	338.7 ± 4.6
3.7–5 + $\psi(1s, 2s)$	143.1 ± 5.4
5–9.3	68.7 ± 1.1
9.3–12	12.1 ± 0.5
12– ∞	18.0 ± 0.1
Total	6294 ± 62

Table 4: Contributions to $a_\mu(\text{Had}; 1)$ from different energy regions as found by Davier and Höcker.[24]

the energy divisions in Tables 3 and 4 do not coincide, one can see indications of improvement throughout Table 4. Particularly significant is the error reductions for $2m_\pi \leq \sqrt{s} \leq 1.8$ GeV due to tau data as well as for $\sqrt{s} \gtrsim 1.8$ GeV due to the use of perturbative QCD (now confirmed by BES data).

4.3 Detailed Discussion of CLY85 and AY

We now discuss some of the specific issues raised by Ref. [25]. Before undertaking this discussion, we note that the evaluations of $a_\mu(\text{Had}; 1)$ in CLY85 and AY95 are somewhat high, but consistent with other analyses, and for some of the same reasons that B85, KNO85, EJ95, etc. are now considered to be obsolete, these two should also be so considered. We also recognize that CLY85 represented the first attempt to use perturbative QCD down to low energies in the evaluation of $a_\mu(\text{Had}; 1)$. While it took some time to be accepted, the use of QCD, along with τ -decay data has resulted in substantial improvements [23, 24]. Nevertheless, we feel compelled by the confusion which has been recently generated by Ref. [25] to detail why we feel that CLY85 and AY95 should not be used in comparison with experiment, at least not with the same conviction as later improved determinations. In fact, we argue that both should be updated before a serious comparison with more up-to-date approaches can be made.

Our first criticism of CLY85 and AY95 involves their treatment of the ρ resonance. Both employ a parametrization of the pion electromagnetic form factor based on Particle Data Group ρ meson parameters, rather than direct data, and they assign a very small uncertainty to their evaluation of the dispersion integral in that important low-energy region. Comparison of their results with more recent data-driven studies suggest that their evaluation of the ρ contribution was too large. Part of the problem may be traced to their use of a relatively low mass for the ρ , 768.5 MeV, and larger width $\Gamma_\rho \simeq 150$ MeV when compared with more recent detailed studies of e^+e^- and tau decay data which suggest $m_\rho \simeq 776$ MeV and $\Gamma_\rho \simeq 146$ MeV. At the very least, the uncertainty inherent in their approach should have been considerably larger. However, if one merely changes the ρ mass and width to the values currently more consistent with data, the AY value of $a_\mu(\text{Had}; 1)$ is (very) roughly lowered by ~ 100 , ($\times 10^{-11}$) a significant reduction. To put it on a level with more data-driven recent studies, it would need to be fully updated.

A second more disturbing problem with the AY95 analysis is its use of perturbative QCD down to very low energies ~ 1.4 GeV and its unconventional treatment of heavy quark thresholds used in conjunction with resonance contributions evaluated by other authors. Rather than comment on the details of their analysis, we consider their results for another dispersion integral,

$$\Delta\alpha_{\text{had}} = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2) - i\epsilon}, \quad (14)$$

the hadronic loop corrections to the fine structure constant $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$. That quantity has been evaluated by many authors[19, 22, 23, 24, 36, 37, 38, 39, 40, 41, 42] because of its critical use in comparing α , m_Z and G_μ with precision measurements of m_W , $\sin^2\theta_W^{\text{eff}}$, etc. In Fig.5, we compare various evaluations of that quantity. Notice, its significant improvement in the more recent evaluations primarily because of improved data. The only calculation which deviates significantly from the others is the value

$$\Delta\alpha_{\text{had}}^{(5)}(m_Z^2) = 289.4 \pm 4.4 \times 10^{-4} \quad (\text{AY95}) \quad (15)$$

given in the text of Adel and Yndurain [21]. (As in the case of hadronic contributions to a_μ , AY95 give very different results for $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$, called $\Delta\alpha_h$, in their text and abstract.)

The deviation of Eq.15 by about 3σ from the central value of more up-to-date evaluations, which give $\sim 276\text{--}277 \times 10^{-4}$ would seem to invalidate the perturbative analysis of heavy quark thresholds used in conjunction with explicit resonance contributions for that quantity (suggesting double counting) and thereby call into question its use for $g_\mu - 2$ studies. We might note that comparison of m_W and $\sin^2\theta_W^{\text{eff}}$ can also be used in the Standard Model framework to obtain (a less precise) $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$ which also disagrees with Eq.15. Again, it seems that one should lower the AY estimate

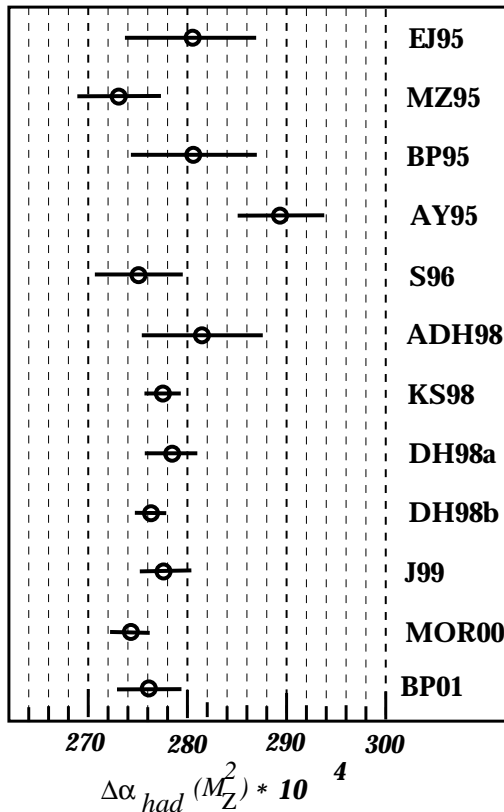


Figure 5: A comparison of the calculated values of $\Delta\alpha_{had}(M_Z^2)$.

of the high energy hadronic contributions to $a_\mu(\text{Had}; 1)$. Taken together with the ρ resonance region reduction mentioned above, such a shift would then move AY's central value down to almost exactly DH98b. Of course, a full revision of AY should actually be made in any serious update of their approach.

Based on the above comparisons, we conclude that although CLY85 and AY95 represent pioneering efforts to incorporate perturbative QCD into a_μ , their results are not state-of-the-art, are not well supported by data and should not be seriously considered on a par with more recent evaluations of $a_\mu(\text{Had}; 1)$. Furthermore, an update of those analyses is likely to significantly reduce their central values to the level very similar to more recent evaluations.

We should also note that several more recent (unpublished) updates or new studies of $a_\mu(\text{Had}; 1)$ beyond those in Table 1 have appeared. Eidelman and Jegerlehner updated their 1995 result with newer data and found [43]

$$6967(119) \times 10^{-11} \quad (\text{EJ99}) \quad (16)$$

and Jegerlehner has further updated that result (from e^+e^- data) to [44]

$$6974(105) \times 10^{-11} \quad (\text{J2000}) \quad (17)$$

A more recent theory-driven analysis (but with parametrization set by e^+e^- and τ decay data) due to Narison found [45]

$$6970(76) \times 10^{-11} \quad (\text{N2001}) \quad (18)$$

All of these results are in good accord with DH98b which was used in comparison with a_μ^{exp} [1]. Any of the up-to-date studies will give a deviation between 2 and 2.6 sigma when compared with experiment.

5 Summary and Conclusions

The QED and weak contributions to $(g-2)$ are known to an accuracy well below what might be accessible to E821. The higher-order hadronic contribution also appears to be under control. The recent use of perturbative QCD along with τ -decay data have greatly improved the uncertainty in the leading hadronic contribution, and supersedes the earlier analyses which only used (more limited) data from electron-positron annihilation to hadrons as input. It is clear from the literature that one needs to be careful when using parameterizations in the important ρ region, unless well supported by data. The use of perturbative QCD at relatively low energies, which was pioneered by CLY85 has led to substantial gains in our understanding, and the agreement of the recent BES data [12] with the QCD predictions of DH98a [23] gives one confidence in the validity of the role of modern QCD studies in this discussion.

The knowledge of the hadronic contribution to a_μ has improved dramatically from the mid 1980s when the $(g-2)$ experiment began. New high quality e^+e^- data have become available from Novosibirsk ($\sqrt{s} = \text{threshold} - 1.4 \text{ GeV}$), and from Beijing ($\sqrt{s} = 2.0 - 5.0 \text{ GeV}$). The entire sample of LEP τ -decay data, as well as the CLEO τ data [27] have also become available. An updated global analysis of all of these data, along with QCD information is underway by Eidelman, Davier and Höcker. These authors have now spent many years working on this topic, and we welcome their combined efforts to produce a new value for $a_\mu(\text{Had}; 1)$. This new more complete analysis should make all of the previous analyses obsolete.

Currently the best published evaluation of $a_\mu(\text{Had}; 1)$ is the work by Davier and Höcker [23, 24]. We see no reason to ignore the substantial amount of additional information which went into these analyses, and rather to use an earlier evaluation which contains only part of our current knowledge. To us, the assertion that one can ignore the latest information, or the claim that earlier, and by necessity, less complete analyses are on an equal footing is incomprehensible, particularly when the earlier approach [21] fails so badly for $\Delta\alpha_{\text{had}}(m_Z^2)$ and has other serious deficiencies. Both in

experimental and theoretical physics, it has been the course of scientific progress to have new information and improved analyses of problems replace the results of earlier efforts.

We are fortunate that at this point several independent authors have considerable experience in the delicate issues which are involved in obtaining a value for $a_\mu(\text{Had}; 1)$. They have brought new insights to the field. We anxiously await their updated evaluation of this important number.

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References

- [1] H.N. Brown, Muon $(g - 2)$ collaboration, Phys. Rev. Lett. **86**, 2227 (2001).
- [2] H.N. Brown *et al.*, $(g - 2)$ Collaboration, Phys. Rev. D **62**, 091101 (2000).
- [3] R.M. Carey *et al.*, $(g - 2)$ Collaboration, Phys. Rev. Lett. **82**, 1632 (1999).
- [4] T. Kinoshita, Rep. Prog. Phys. **59**, 1459 (1996).
- [5] V.W. Hughes and T. Kinoshita, Rev. Mod. Phys. **71**, S133 (1999).
- [6] A. Czarnecki and W. Marciano, Nucl. Phys. B (Proc. Suppl.) **76**, 245 (1999).
- [7] A. Czarnecki, B. Krause and W.J. Marciano, Phys. Rev. D **52**, R2619 (1995).
- [8] A. Czarnecki, B. Krause and W.J. Marciano, Phys. Rev. Lett. **76**, 3267 (1996).
- [9] G. Degrossi and G.F. Giudice, Phys. Rev. D **58**, 53007 (1998).
- [10] A. Czarnecki and W. Marciano, hep-ph/0102122, (2001).
- [11] R.R.Akhmetshin et al., Phys. Lett. B **475**, 190 (2000), R.R.Akhmetshin et al., hep-ex/9904027, and R.R.Akhmetshin et al., Nucl. Phys. A **675**, 424 (2000) and references therein.
- [12] J.Z. Bai, *et al.*, BES Collaboration, Phys. Rev. Lett. **84**, 594 (2000), and reported at XXXth ICHEP in Osaka by Zheng-guo Zhao.
- [13] See transparency #20 from the talk by A. Höcker at XXXVI Rencontres de Moriond, <http://moriond.in2p3.fr/EW/2001/program.html> for a figure with all the $R(s)$ data including the BES results.

- [14] T. Kinoshita and W.J. Marciano in *Quantum Electrodynamics* (Directions in High Energy Physics, Vol. 7), ed. T. Kinoshita, (World Scientific, Singapore, 1990), p. 419.
- [15] L. Barkov, *et al.*, Nucl. Phys. B **256**, 365 (1985).
- [16] T. Kinoshita, B. Nizić and Y. Okamoto, Phys. Rev. D **31**, 2108 (1985).
- [17] A. Casas, C. Lopez, and F.J. Yndurain, Phys. Rev. D **32**, 736 (1985).
- [18] Ľubomír Martinovič and Stansilav Dubníka, Phys. Rev. D **42**, 884 (1990).
- [19] S. Eidelman and F. Jegerlehner, Z. Phys. C **67**, 585 (1995).
- [20] D.H. Brown and W.A. Worstell, Phys. Rev. D **54**, 3237 (1996).
- [21] K. Adel and F.J. Yndurain, hep-ph/9509378, which is printed in Rev. Acad. Ciencias (Esp.) 92, (1998). We do not have access to the latter reference.
- [22] R. Alemany, M. Davier and A. Höcker, Eur. Phys. J. C **2**, 123 (1998).
- [23] M. Davier and A. Höcker, Phys. Lett. B **419**, 419 (1998).
- [24] M. Davier and A. Höcker, Phys. Lett. B **435**, 427 (1998).
- [25] F.J. Yndurain, hep-ph/0102312, (2001).
- [26] G. Gounaris, J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968).
- [27] S. Anderson *et al.*, CLEO Collaboration, Phys. Rev. D **61**, 112002 (2000).
- [28] D.H. Brown, private communication, says that they used the data of D. Bisello, *et al.*, LAL preprint: 90-35 (1990).
- [29] Bernd Krause, Phys. Lett. B **390**, 392 (1997).
- [30] M.B. Einhorn, Phys. Rev. D **49** (1993) 1668.
- [31] M. Hayakawa, T. Kinoshita and A.I. Sanda, Phys. Rev. Lett. **75**, 790 (1995), and Phys. Rev. D **D54**, 3137 (1996).
- [32] Johan Bijnens, Elisabetta Pallante and Joaquim Prades, Phys. Rev. Lett. **75**, 1447 (1995), Phys. Rev. Lett. **75**, 3781 (1995) and Nucl. Phys. B **474**, 379 (1996).
- [33] M. Hayakawa and T. Kinoshita, Phys. Rev. D **57**, 465 (1998).
- [34] See M. Hayakawa PRD **54**, R6586 (1996).

- [35] T. Kinoshita, private communication.
- [36] A.D. Martin and D. Zeppenfeld, Phys. Lett. B **345**, 558 (1995).
- [37] H. Burkhardt and B. Pietrzyk, Phys. Lett. B **356**, 398 (1995).
- [38] M. Swartz, Phys. Rev. D **53**, 5268 (1996).
- [39] J.H. Kühn and M. Steinhauser, Phys. Lett. B **437**, 425 (1998).
- [40] F. Jegerlehner, hep-ph/0001386, Presented at the IV Int. Symp. on Radiative Corrections, 1998.
- [41] A.D. Martin, J. Outhwaite and M.G. Ryskin, hep-ph/0012yyyy 2001.
- [42] H. Burkhardt and B. Pietrzyk, LAPP-EZP 2001-03, submitted to Phys. Lett. February 2001.
- [43] S. Eidelman and F. Jegerlehner, 1999.
- [44] F. Jegerlehner 2000, hep-ph/0104304.
- [45] Stephan Narison, hep-ph/0103199, March 2001.